Symmetries in 2HDM and beyond

Lecture 1: Describing 2HDM efficiently

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General outline

- Lecture 1: describing 2HDM efficiently
- Lecture 2: symmetries in 2HDM
- Lecture 3: abelian symmetries in bSM models
- Lecture 4: non-abelian symmetries in NHDM

Nano-glossary:
2HDM = two-Higgs-doublet model
NHDM = N-Higgs-doublet model
bSM = beyond the Standard Model
EWSB = electroweak symmetry breaking

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Question

Q0.1: count how many questions you encounter during lectures.

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- References: I will provide only few references; on 2HDM, there exists big review: *[REVIEW]* $\equiv$ *[Branco, Ferreira, Lavoura, Rebelo, Sher, Silva, 2012]*.

Outline of lecture 1

1. 2HDM in a nutshell
2. Basis-independent methods in 2HDM
3. Bilinears and geometric picture

Higgs mechanism in 2HDM

Two-Higgs-doublet model (2HDM) suggested by T. D. Lee in 1973: two Higgs doublets, 8 real fields in total:

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}.$$

Upon EWSB: three scalars absorbed by massive gauge bosons, 5 physical Higgses remain: two charged ($H^\pm$) and three neutral (usually h, H, A).

Physics motivation:

- MSSM contains two Higgs doublets.
- Richer dynamics of EW symmetry breaking; possibility of *CP*-violation coming purely from the Higgs sector.
- various astroparticle consequences.

2HDM: characteristic features

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$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2 e^{i\xi}\end{pmatrix}.$$

  Both doublets couple to gauge bosons → $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV is fixed. The ratio $\tan\beta = \frac{v_2}{v_1}$ can be variable.

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  Both doublets couple to gauge bosons → $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV is fixed. The ratio $\tan\beta = \frac{v_2}{v_1}$ can be variable.
- The two (complex) vevs can have a relative phase ξ, which leads to *CP*-violation. This can happen spontaneously after EWSB, you don't need to put the phase by hand! This was the original motivation of T. D. Lee in 1973 (at that time, only three quarks were known, and the source of *CP*-violation was enigmatic).

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- Intricate phase transitions in early Universe → cosmological consequences.

Analyzing 2HDM

How do we analyze 2HDM?

1. Scalar sector: write down the Higgs potential, find the global minimum, expand potential, define the physical Higgses.
2. Fermions: insert vevs to Yukawa terms, determine quark and lepton masses and mixing, adjust all parameters to reproduce the observed values.
3. Pheno: expand the lagrangian in terms of physical Higgses, study the resulting phenomenology, study the new Higgs-induced corrections to SM processes.

Analyzing 2HDM

The most general Higgs potential:

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \mathrm{h.c.}\right] \end{aligned}$$

It contains $4 + 10 = 14$ free parameters, instead of just two for the SM Higgs potential. Huge parameter space!

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- not clear how to describe the entire parameter space,
- not clear which parameters are responsible for what, which parameters are more important and which are redundant.

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What's left?

- Working with simplified models only? Might be losing interesting effects in the "bulk" or in the corners of the parameter space.
- Resorting to numerical methods? Blindly scanning parameter space? Awkward: lack of robust analytic conditions for positivity, charge-breaking/neutral minimum, inability to find all accidental symmetries, huge redundancy of work due to basis dependence...
- Or is there any other method available?

Analyzing 2HDM

Yes!

There exists a method to qualitatively analyze the most general 2HDM scalar sector. If does not give the exact position of the global minimum, but it gives

- analytic conditions for: boundedness from below, charge-breaking/neutral minimum, explicit or spontaneous *CP*-violation;
- list of all accidental symmetries possible in 2HDM scalar sector, analytic conditions when symmetries arise and how they break,
- the number of minima and criteria for their coexistence,

Two examples of symmetries

$\mathbb{Z}_2$ symmetry: $\phi_1 \to \phi_1,\ \phi_2 \to -\phi_2$

$$\begin{aligned} V_{\mathbb{Z}_2} &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) \cancel{+ m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)}\right] \\ &+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \left[\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \cancel{\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2)} + \cancel{\lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2)} + \mathrm{h.c.}\right] \end{aligned}$$

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\end{aligned}
$$

S_2 symmetry: $\phi_1 \leftrightarrow \phi_2$

$$
\begin{aligned}
V_{S_2} &= -\frac{1}{2}m_{11}^2\Big[(\phi_1^\dagger\phi_1) + (\phi_2^\dagger\phi_2)\Big] - \frac{1}{2}m_{12}^2\Big[(\phi_1^\dagger\phi_2) + (\phi_2^\dagger\phi_1)\Big] \\
&+ \frac{\lambda_1}{2}\Big[(\phi_1^\dagger\phi_1)^2 + (\phi_2^\dagger\phi_2)^2\Big] + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
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Lesson: blind case-by-case checking is a redundant work.

If we don't want to study the same model over and over and over again! We want to study essentially different models.

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- In general the relation can be highly non-evident!

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \to U \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \quad \text{or} \quad \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \to U \begin{pmatrix}\phi_1^*\\ \phi_2^*\end{pmatrix}, \quad U \in U(2)\,.$$

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Anti-unitary transformations = generalized CP transformations.

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- Start with a general 2HDM with some m's and λ's and perform a Higgs basis change. We obtain another 2HDM with new m's and λ's $\to$ reparametrization of the Higgs potential.

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There…

Anti-… …ns.

- Start … …rm a Higgs … … and λ's

$\rightarrow$ re…

Question

Q1.1: construct the generic 2HDM potential symmetric under the following gCP transformation:

$$\phi_1 \to \phi_2^*\,, \quad \phi_2 \to \phi_1^*\,,$$

and not symmetric under anything else.

The same question but for

$$\phi_1 \to \phi_2\,, \quad \phi_2 \to -\phi_1\,.$$

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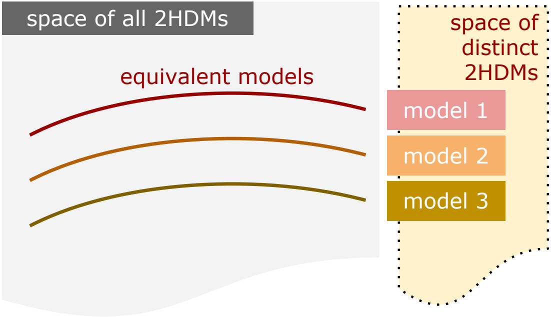
Basis dependence

We just discovered that the space of 2HDMs has certain structure.



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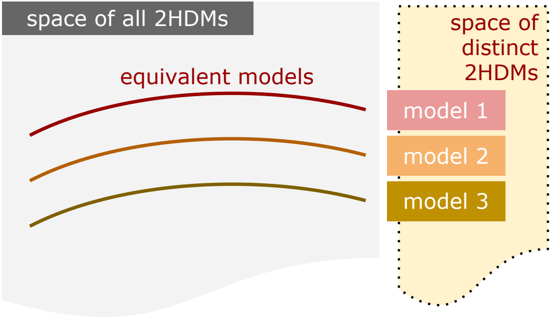

Question

Q1.2: What are the dimensionalities of the space of all 2HDMs and of the space of distinct 2HDMs?

Basis invariant methods

Compact form of the 2HDM potential *[Botella, Silva, 1995; REVIEW]*:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,, \quad a,b,c,d = 1,2\,.$$

Basis change $\phi_a \to U_{aa'}\phi_{a'}$ induces reparametrization:

$$Y_{ab} \to U_{aa'}U^*_{bb'}Y_{a'b'}\,, \quad Z_{ab,cd} \to U_{aa'}U^*_{bb'}U_{cc'}U^*_{dd'}Z_{a'b',c'd'}\,.$$

but fully contracted tensors such as

$$Y_{aa}\,, \quad Z_{ab,ba}\,, \quad Z_{ab,bc}Y_{ca}\,, \quad Z_{ab,cd}Z_{dc,ef}Z_{fe,ba}\,, \quad \dots$$

do not change → basis-independent quantities.
Detailed theory developed in *[Davidson, Haber, 2005; Gunion, Haber, 2005]*:

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Unfortunately, solving these tasks is very difficult and non-intuitive.
An example: parameters m_{ij} and λ_i can be complex; yes, there might exist a basis when they all become real (this is important for explicit CP-conservation). What is the basis-independent marker of this situation?

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Investigated in *[Gunion, Haber, 2005]*:

$$\begin{aligned} &\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \quad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,, \\ &\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,, \\ &\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,. \end{aligned}$$

Found by *Mathematica* search among millions of possible variants.

Bilinear formalism

Luckily, there exists a much simpler and more intuitive way of finding basis-independent quantities — geometric method in the space of bilinears.

Developed independently and in different aspects by three groups:
[Maniatis, von Manteuffel, Nachtmann, Nagel, 2004–2007]:
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I will show a simplified version of this formalism, which does not use all the reparametrization freedom available.

Bilinear formalism

The Higgs potential is built of gauge-invariant bilinears $(\phi_a^\dagger \phi_b)$ (EW-orbits). Let's organize them into combinations:

$$r_0 = \phi_a^\dagger \phi_a \equiv (\phi_1^\dagger \phi_1) + (\phi_2^\dagger \phi_2)\,,$$
$$r_i = \phi_a^\dagger \sigma^i_{ab} \phi_b \equiv \begin{pmatrix} 2\mathrm{Re}(\phi_1^\dagger \phi_2) \\ 2\mathrm{Im}(\phi_1^\dagger \phi_2) \\ (\phi_1^\dagger \phi_1) - (\phi_2^\dagger \phi_2) \end{pmatrix}.$$

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Basis change: r_0 invariant, r_i transforms by an $SO(3)$ rotation.
Anti-unitary transformation: r_0 invariant, r_i transforms by an improper rotation (reflection and $SO(3)$ rotation).

This is the well-known correspondence between groups $SU(2)$ and $SO(3)$.

Orbit space

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From definitions: $r_0 \geq 0$, $r_0^2 - r_i^2 = 4\left[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right] \geq 0$.

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Question

Q1.3: prove it!

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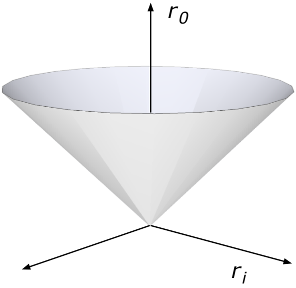
This transition maps the entire space of Higgs fields ϕ_a to the orbit space in terms of $(r_0,\ r_i)$. What's the shape of the orbit space?

From definitions: $r_0 \geq 0,\ r_0^2 - r_i^2 = 4\left[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right] \geq 0.$

The orbit space is the surface and interior of the "future lightcone" in the $(r_0,\ r_i)$ space.



The analogy with the space-time $r^\mu = (r_0,\ r_i)$ is deeper than it might seem. The orbit space indeed has the Minkowski space structure; the Lorentz group $SO(1,3)$ can be induced by basis changes. *[Ivanov, 2007].*

Orbit space

Three kinds of points on the cone → three kinds of minima:

- interior: doublets ϕ_1 and ϕ_2 are not proportional to each other:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix}$$

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Question

Q1.4: prove it!

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  → charge-breaking vacuum;
- surface: doublets ϕ_1 and ϕ_2 are proportional → neutral vacuum;
- apex: $(r_0, r_i) = 0$ → EW-symmetric vacuum.

Natural physical interpretation of geometric properties.

Higgs potential

The Higgs potential of the general 2HDM now becomes

$$V = -M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu = -(M_0 r_0 - M_i r_i) + \frac{1}{2}\left(\Lambda_{00} r_0^2 - 2\Lambda_{0i} r_0 r_i + \Lambda_{ij} r_i r_j\right) .$$

All 14 free parameters are placed in 4 components of M_μ and 10 components of $\Lambda_{\mu\nu}$.

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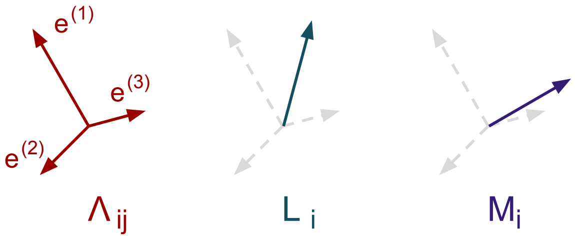
$$M_\mu = \frac{1}{4}\left(m_{11}^2 + m_{22}^2,\ -2\mathrm{Re}\, m_{12}^2,\ 2\mathrm{Im}\, m_{12}^2,\ -m_{11}^2 + m_{22}^2\right) ,$$

$$\Lambda_{\mu\nu} = \frac{1}{2}\begin{pmatrix} \frac{\lambda_1+\lambda_2}{2} + \lambda_3 & -\mathrm{Re}(\lambda_6+\lambda_7) & \mathrm{Im}(\lambda_6+\lambda_7) & -\frac{\lambda_1-\lambda_2}{2} \\ -\mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4 + \mathrm{Re}\lambda_5 & -\mathrm{Im}\lambda_5 & \mathrm{Re}(\lambda_6-\lambda_7) \\ \mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}\lambda_5 & \lambda_4 - \mathrm{Re}\lambda_5 & -\mathrm{Im}(\lambda_6-\lambda_7) \\ -\frac{\lambda_1-\lambda_2}{2} & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \frac{\lambda_1+\lambda_2}{2} - \lambda_3 \end{pmatrix}$$

Higgs potential

An 2HDM is defined by: two scalars M_0, Λ_{00}, two 3-vectors M_i and $L_i = \Lambda_{0i} r_0$, and a symmetric tensor Λ_{ij}.



- Λ_{ij} is defined by its eigenvectors $e_i^{(k)}$, which define a natural basis in the r_i space, and its eigenvalues Λ_k;
- M_i and L_i are defined by their orientation in this natural basis.
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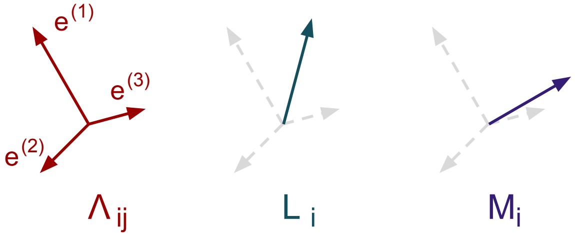

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Direct road to basis-independent quantities!

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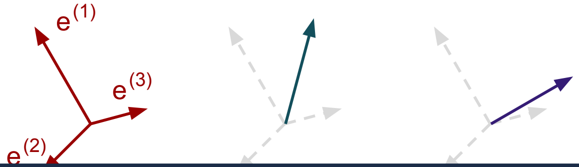

Question

Q1.5: derive all algebraically independent basis-invariant expressions which are linear or quadratic in λ_i.

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  All this can be established without knowing the exact position of the global minimum.
- The entire phase diagram of the 2HDM scalar potential at the tree-level can be explicitly constructed, with all its critical points, surfaces of phase transition, etc.

Lecture 1 Summary

- 2HDM is a reasonably conservative framework for bSM model building. Yet, it is very rich in physical consequences and offers large space of free parameters. Today, it is a standard reference model for the bSM physics.
- The relatively simple structure of 2HDM scalar sector suggests that it should be explored in its most general form and in full detail. But — the straightforward approach miserably fails even at the first step $\to$ more efficient approaches are needed!
- There are two powerful approaches: the tensorial method and the geometric method. The latter, through some fine geometric and group-theoretic results, can answer all structural questions about 2HDM scalar sector.