INDUCED HAMILTONIAN MAPS ON THE SYMPLECTIC QUOTIENT

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Abstract. Let (M,ω) be a symplectic manifold and G a compact Lie group that acts on M. Assume that the action of G on M is Hamiltonian. Then a G-equivariant Hamiltonian map on M induces a map on the symplectic quotient of M by G. Consider an autonomous Hamiltonian with compact support on M, with no non-constant closed trajectory in time less than 1 and time-1 map f. If the map f^H descends to the symplectic quotient to a map $\Phi(f^H)$ and the symplectic manifold M is exact and $\mathrm{Ham}(M,\omega)$ has no short loops, we prove that the Hofer norm of the induced map $\Phi(f^H)$ is bounded above by the Hofer norm of f^H.

1. Introduction

Consider (M,ω) a symplectic manifold and G a compact Lie group that acts on M. Assume that the action of G on M is Hamiltonian and let $\mu : M \longrightarrow \mathfrak{g}^*$ be the moment map that corresponds to the Hamiltonian action. Assume that μ is a proper map and that $0 \in \mathfrak{g}^*$ is a regular value of μ. Thus $K := \mu^{-1}(0)$ is a compact G-equivariant submanifold of M. Denote by $\mathrm{Ham}^c_G(M,K)$ the set of G-equivariant Hamiltonian maps of M with compact support that map the compact G-submanifold K to itself. Let $\mathrm{Ham}^c_G(M,K)_0$ be the connected component that contains the identity map.

Denote by $(M/\!\!/G,\omega_{\mathrm{red}})$ the symplectic reduction of the Hamiltonian G-manifold M. Every Hamiltonian map in $\mathrm{Ham}^c_G(M,K)_0$ induces a diffeomorphism on the quotient $M/\!\!/G$. In Sec. 3, we prove that the induced map is a Hamiltonian map on the symplectic quotient $(M/\!\!/G,\omega_{\mathrm{red}})$. Thus there is a well defined group homomorphism

$$\Phi : \mathrm{Ham}^c_G(M,K)_0 \longrightarrow \mathrm{Ham}(M/\!\!/G,\omega_{\mathrm{red}}).$$

Moreover in Prop. 3.6 we prove that Φ is surjective group homomorphism. Hence the map Φ can be thought as a Kirwan map.

The purpose of this article is to obtain a relation between the Hofer norm of f and the Hofer norm of the induced map $\Phi(f)$. We achieve this for a special kind of Hamiltonian maps. We denote the Hofer norm of a Hamiltonian map f by $\|f\|$.

2000 *Mathematics Subject Classification.* Primary: 53D20, 57R17; Secondary: 57R50.
Key words and phrases. Hamiltonian group, Hofer's norm, Symplectic reduction.
The author was supported by a PROMEP grant.

Theorem 1.1. *Consider* (M,ω) *an exact symplectic manifold such that* $\mathrm{Ham}(M,\omega)$ *has no short loops. Let* G *be a compact Lie group that acts in a Hamiltonian way on* M *with* μ *and* K *as above; and* $H : M \longrightarrow \mathbb{R}$ *an autonomous Hamiltonian with compact support and time-1 map* f^H*. If the Hamiltonian isotopy generated by* H *lies in* $\mathrm{Ham}^c_G(M,K)_0$ *and has no non-constant trajectory in time less than 1, then*

$$\|\Phi(f^H)\| \leq \|f^H\|.$$

This means that the energy of the induced map $\Phi(f)$ is less than the energy of f. There are many maps for which the inequality of Thm. 1.1 is a strict inequality. If $f \in \mathrm{Ham}^c_G(M,K)_0$ is different from the identity map and the support of f is disjoint from the G-submanifold K, then $0 = \|\Phi(f)\| < \|f\|$. The hypothesis on the function H of Thm. 1.1 is not too restrictive. In Lemma 3.4 we give sufficient conditions that guarantee that the Hamiltonian isotopy lies in the subgroup $\mathrm{Ham}^c_G(M,K)_0$ by analyzing the Hamiltonian vector field X_H on M.

Is important to note that Thm 1.1 is in part a consequence of the fact that for such autonomous Hamiltonian H, the Hofer norm of f^H equals the length of the isotopy induced by H. (See Thm. 2.2 below.) This result was proved by Bialy-Polterovich [3] and Hofer [4] for $\mathbb{R}^{2n}$, and later generalized by Lalonde-McDuff [5] for exact symplectic manifolds such that $\mathrm{Ham}(M,\omega)$ has no short loops.

The author thanks Dusa McDuff for pointing out an inaccuracy in an earlier draft of this article, and also the referee for very helpful comments and a simplification in the proof of Lemma 3.5.

2. Symplectic reduction and the Hofer metric

Let (M,ω) be a symplectic manifold and G a compact Lie group that acts on M in a Hamiltonian way. Denote by $\mu : M \longrightarrow \mathfrak{g}^*$ the moment map associated with the Hamiltonian G-action. Assume that μ is a proper map, $0 \in \mathfrak{g}^*$ is regular value of μ, and that G acts freely on K. So defined, K is a smooth compact G-submanifold of M. Write $M/\!\!/G$ for the quotient of K by the action of G.

If $i : K \hookrightarrow M$ denotes the inclusion map and $\pi : K \longrightarrow M/\!\!/G$ the projection map of the G-bundle, then there is a unique symplectic form ω_{red} on $M/\!\!/G$ such that

$$i^*(\omega) = \pi^*(\omega_{\mathrm{red}}).$$

The symplectic manifold $(M/\!\!/G, \omega_{\mathrm{red}})$ is called the symplectic quotient of the Hamiltonian G-manifold (M,ω). For more details see [1] and [6].

We comment about the tangent plane of K at a point, since this will be useful later on. For the details see ([6], p. 169). Let X_ξ be the vector field on M induced by the action of G and the vector $\xi \in \mathfrak{g}$. For $p \in K$, denote by $\mathcal{O}(p)$ the orbit of p in M. Then $T_p\mathcal{O}(p)$ is generated by the induced vectors $X_\xi(p)$, $\xi \in \mathfrak{g}$. Moreover

$$T_pK = (T_p\mathcal{O}(p))^\omega.$$

Here $(T_p\mathcal{O}(p))^\omega$ stands for the ω-complement subspace of $T_p\mathcal{O}(p)$ in T_pM.

Briefly we discuss the Hofer norm of a Hamiltonian map. An isotopy $\{f_t\}_{0\leq t\leq 1}$ of symplectic maps that starts at the identity, $f_0 = \mathrm{id}_M$, induces a unique time-dependant vector field X_t given by the equation

$$\frac{d}{dt} f_t = X_t \circ f_t.$$

The isotopy $\{f_t\}$ is said to be a Hamiltonian isotopy if there is a time-dependant smooth function $F_t : M \longrightarrow \mathbb{R}$, called the Hamiltonian function, such that

$$\iota(X_t)\omega = -dF_t.$$

The group $\mathrm{Ham}(M,\omega)$ of Hamiltonian transformations is defined as the set of time-1 maps f_1, of Hamiltonian isotopies $\{f_t\}$ that start at the identity map.

The length of a Hamiltonian isotopy is defined as follows. Let F_t be a Hamiltonian function that corresponds to the Hamiltonian isotopy $\{f_t\}$. Note that F_t is unique up to an additive constant. The length of a Hamiltonian isotopy $\{f_t\}$ is defined as

$$\mathcal{L}(\{f_t\}) = \int_0^1 \sup_{p\in M} F_t(p) - \inf_{p\in M} F_t(p) \; dt.$$

The Hofer norm of f, written $\|f\|$, is defined as the infimum of $\mathcal{L}(f_t)$ over all Hamiltonian isotopies that start at id_M and end at f. For a detailed introduction to the Hofer norm see [7].

A Hamiltonian map $f \in \mathrm{Ham}(M,\omega)$ is called autonomous if there is a Hamiltonian isotopy $\{f_t\}$ such that $f_0 = \mathrm{id}_M$, $f_1 = f$, and the Hamiltonian function induced by the isotopy $\{f_t\}$ is time independent. An important fact of autonomous Hamiltonian maps for this article is the following. In [2], Banyaga proved that when the symplectic manifold (M,ω) is compact and $H^1(M,\mathbb{R}) = 0$, then set of autonomous Hamiltonian maps generate the group $\mathrm{Ham}(M,\omega)$ of Hamiltonian maps of (M,ω).

There are some cases when a Hamiltonian function induces a length-minimizing isotopy. This is the case of the next result. A symplectic manifold (M,ω) is called exact if $\omega = d\lambda$ for some 1-form λ; and $\mathrm{Ham}(M,\omega)$ has no short loops if 0 is an isolated point of the image of $\mathcal{L}$. For example $\mathbb{R}^{2n}$ with the standard symplectic form is exact and $\mathrm{Ham}^c(\mathbb{R}^{2n},\omega_0)$ has no short loops.

Theorem 2.2 (Lalonde-McDuff [5]). *Let (M,ω) be an exact symplectic manifold such that $\mathrm{Ham}(M,\omega)$ has no short loops. If $H : M \longrightarrow \mathbb{R}$ is an autonomous Hamiltonian with compact support and time-1 map f^H such that the induced isotopy $\{f_t^H\}$ has no non-constant trajectory in time less than 1, then*

$$\|f^H\| = \mathcal{L}(\{f_t^H\}) = \sup_{x\in M} H(x) - \inf_{x\in M} H(x).$$

3. Equivariant maps and induced maps on the symplectic quotient

A symplectomorphism $f \in \mathrm{Symp}(M,\omega)$ is said to be G-equivariant if

$$f(g.p) = g.f(p)$$

for all $g \in G, p \in M$. Denote by $\mathrm{Symp}^c_G(M,\omega)$ the set of G-equivariant symplectomorphisms of (M,ω) with compact support. And by $\mathrm{Ham}^c_G(M,\omega)$ the group of G-equivariant Hamiltonian maps,

$$\mathrm{Ham}^c_G(M,\omega) = \mathrm{Ham}^c(M,\omega) \cap \mathrm{Symp}^c_G(M,\omega).$$

Let $f \in \mathrm{Symp}^c_G(M,\omega)$ and assume that f maps K to itself. Then f induces a diffeomorphism $\tilde{f}$ on the symplectic quotient $M/\!\!/G$, defined by the commutative diagram

$$\begin{array}{ccc} K & \xrightarrow{f|_K} & K \\ {\scriptstyle\pi}\downarrow & & \downarrow{\scriptstyle\pi} \\ M/\!\!/G & \xrightarrow[\tilde{f}]{} & M/\!\!/G \end{array}$$

That is $\pi \circ f|_K = \tilde{f} \circ \pi$.

Lemma 3.3. *Let $f \in \mathrm{Symp}^c_G(M, \omega)$ such that it maps the G-submanifold K to itself. Then the induced map $\tilde{f}$ is a symplectomorphism of $(M/\!\!/G, \omega_{\mathrm{red}})$.*

Proof. From the above diagram we have, $(f|_K)^* \circ \pi^* = \pi^* \circ \tilde{f}^*$. Therefore

$$\begin{aligned} \pi^* \circ \tilde{f}^*(\omega_{\mathrm{red}}) &= (f|_K)^* \circ \pi^*(\omega_{\mathrm{red}}) \\ &= (f|_K)^* \circ i^*(\omega) \\ &= i^*(\omega). \end{aligned}$$

Thus $\pi^*(\tilde{f}^*(\omega_{\mathrm{red}})) = \pi^*(\omega_{\mathrm{red}})$. Recall that π is the projection map of a principal bundle. Thus π^* is injective. It follows that $\tilde{f}^*(\omega_{\mathrm{red}}) = \omega_{\mathrm{red}}$ and $\tilde{f}$ is a symplectomorphism of $(M/\!\!/G, \omega_{\mathrm{red}})$. □

Write $\mathrm{Ham}^c_G(M, K)$ for the group of G-equivariant Hamiltonian maps with compact support that map the G-submanifold K to itself. To avoid cumbersome notation we omit the symplectic form from the notation. Let $\mathrm{Ham}^c_G(M, K)_0$ be the connected component that contains the identity map.

Note that for each $f \in \mathrm{Ham}^c_G(M, K)_0$ there is a Hamiltonian isotopy $\{f_t\}_{0\leq t\leq 1}$ in $\mathrm{Ham}^c_G(M, K)$ such that $f_0 = \mathrm{id}_M$ and $f_1 = f$. Thus there is a vector field X_t on M and Hamiltonian function $F_t : M \longrightarrow \mathbb{R}$. Since the isotopy $\{f_t\}$ consists of G-equivariant maps and the symplectic form ω is invariant under the action of G, it follows that the vector field X_t and the function F_t are G-invariant. That is

$$g_*(X_t(p)) = X_t(g.p) \quad \text{and} \quad F_t(g.p) = F_t(p).$$

for all $g \in G$ and $p \in M$. Also, since each f_t maps the G-submanifold K to itself, it follows by the definition of the vector field X_t that $X_t(p) \in T_pK$ for any $p \in K$.

Finally note that the tangent space to $\mathrm{Ham}^c_G(M, K)$ at the identity map consists of Hamiltonian vector fields X with compact support which are G-invariant and $X(p) \in T_pK$ for all $p \in K$. Since $T_pK = (T_p\mathcal{O}(p))^\omega$, then $T_{\mathrm{id}}\mathrm{Ham}^c_G(M, K)$ consists of compactly supported Hamiltonian vector fields X, such that

$$L_{X_\xi}(X) = 0 \quad \text{and} \quad \omega(X_\xi, X)(p) = 0 \tag{1}$$

for all $p \in K$, and $\xi \in \mathfrak{g}$. Here L_{X_ξ} stands for the Lie derivative with respect to X_ξ. Therefore is easy to verify if a Hamiltonian isotopy in $\mathrm{Ham}^c(M, \omega)$ lies in the subgroup $\mathrm{Ham}^c_G(M, K)_0$.

Lemma 3.4. *Let $H : M \longrightarrow \mathbb{R}$ be an autonomous Hamiltonian with compact support and corresponding vector field X_H. Then the isotopy induced by H lies in $\mathrm{Ham}^c_G(M, K)_0$ if X_H satisfies*

$$L_{X_\xi}(X_H) = 0 \quad \textit{and} \quad \omega(X_\xi, X_H)(p) = 0$$

for all $\xi \in \mathfrak{g}$ and $p \in K$.

The previous result shows that it is easy to obtain a Hamiltonian function that satisfies the hypothesis of Thm. 1.1.

Lemma 3.5. *Let $f \in \mathrm{Ham}^c_G(M,K)_0$, then the induced map $\tilde{f}$ is a Hamiltonian map of $M/\!\!/G$.*

Proof. Since f is in $\mathrm{Ham}^c_G(M,K)_0$, there is a Hamiltonian isotopy $\{f_t\}_{0\leq t\leq 1}$ that starts at the identity map and ends at f. From Lemma 3.3, $\{\tilde{f}_t\}_{0\leq t\leq 1}$ is an isotopy of symplectic maps on $M/\!\!/G$ from the identity map to $\tilde{f}$. We will show that the isotopy $\{\tilde{f}_t\}$ is a Hamiltonian isotopy.

Let X_t be the vector field on M and $F_t : M \longrightarrow \mathbb{R}$ the Hamiltonian function induced by the Hamiltonian isotopy $\{f_t\}$. Then X_t is a G-invariant vector field on K. Since $\pi : K \longrightarrow M/\!\!/G$ is a principal G-bundle and X_t is G-invariant, there is a smooth vector field $\tilde{X}_t$ on $M/\!\!/G$ that is π-related to X_t. That is $\pi_*(X_t) = \tilde{X}_t \circ \pi$. We will show that the vector field $\tilde{X}_t$ corresponds to the induced isotopy $\{\tilde{f}_t\}$.

For $p \in K$,

$$X_t(p) = \left.\frac{d}{ds}\right|_{s=0} f_{t+s} \circ f_t^{-1}(p).$$

Then

$$\begin{aligned}
\tilde{X}_t(\pi(p)) = \pi_*(X_t(p)) &= \pi_*\left(\left.\frac{d}{ds}\right|_{s=0} f_{t+s} \circ f_t^{-1}(p)\right) \\
&= \left.\frac{d}{ds}\right|_{s=0} \pi \circ f_{t+s} \circ f_t^{-1}(p) \\
&= \left.\frac{d}{ds}\right|_{s=0} \tilde{f}_{t+s} \circ \pi \circ f_t^{-1}(p) \\
&= \left.\frac{d}{ds}\right|_{s=0} \tilde{f}_{t+s} \circ \tilde{f}_t^{-1}(\pi(p)).
\end{aligned}$$

This means that the vector field $\tilde{X}_t$ is induced by the isotopy $\{\tilde{f}_t\}$.

The G-invariant function F_t, restricted to K, induces a unique smooth function $\tilde{F}_t : M/\!\!/G \longrightarrow \mathbb{R}$ that satisfies $F_t = \tilde{F}_t \circ \pi$.

$$\begin{array}{ccc}
K & & \\
\Big\downarrow \pi & \searrow^{F_t} & \\
M/\!\!/G & \xrightarrow[\tilde{F}_t]{} & \mathbb{R}
\end{array}$$

Finally, equations $\pi_*(X_t) = \tilde{X}_t \circ \pi$, $F_t = \tilde{F}_t \circ \pi$, and $\iota(X_t)i^*\omega = -d(F_t \circ i)$ imply that $\iota(\tilde{X}_t)\omega_{\mathrm{red}} = -d\tilde{F}_t$. That is, $\tilde{X}_t$ is Hamiltonian vector field with Hamiltonian function $\tilde{F}_t$. Thus $\{\tilde{f}_t\}_{0\leq t\leq 1}$ is a Hamiltonian isotopy and $\tilde{f}_1 = \tilde{f} \in \mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}})$. □

It follows from Lemma 3.5, that we have a well-defined group homomorphism

$$\Phi : \mathrm{Ham}^c_G(M,K)_0 \longrightarrow \mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}}).$$

Remark. From the proof of Lemma 3.5 is important to single out that under Φ, Hamiltonian isotopies in $\mathrm{Ham}^c_G(M,K)_0$ are mapped to Hamiltonian isotopies in $\mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}})$. Moreover if $\{f_t\}$ is an isotopy in $\mathrm{Ham}^c_G(M, K)_0$ with corresponding vector field X_t and Hamiltonian function F_t, then the induced isotopy $\{\tilde{f}_t\}$ in $\mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}})$ has the vector field $\tilde{X}_t$ Hamiltonian function $\tilde{F}_t$, such that $\tilde{X}_t$ and X_t are π-related and $F_t = \tilde{F}_t \circ \pi$ on K.

Fix a connection on the principal G-bundle $\pi : K \longrightarrow M/\!\!/G$. Then any smooth vector field $\tilde{Y}$ on $M/\!\!/G$ has a unique lift to K. That is, a vector field Y on K which is G-invariant, horizontal and is π-related to $\tilde{Y}$. The latter means that

$$\pi_*(Y) = \tilde{Y} \circ \pi.$$

Proposition 3.6. *Assume that* $H^1(M/\!\!/G; \mathbb{R}) = 0$ *and* $H^1_c(M, K; \mathbb{R}) = 0$. *Then the group homomorphism*

$$\Phi : \mathrm{Ham}^c_G(M, K)_0 \longrightarrow \mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}}).$$

is surjective.

Proof. Let $h : M/\!\!/G \longrightarrow M/\!\!/G$ be a Hamiltonian map that corresponds to the time-1 map of a time independent Hamiltonian function $H : M/\!\!/G \longrightarrow \mathbb{R}$. We will show that h is in the image of Φ.

Let $\tilde{X}_H$ be the vector field on $M/\!\!/G$ defined by $\iota(\tilde{X}_H)\omega_{\mathrm{red}} = -dH$. Thus h is the time-1 map of the 1-parameter group induced by $\tilde{X}_H$. Let X be the G-invariant horizontal lift of $\tilde{X}_H$ to K. Extend X to all M as a G-invariant vector field with compact support. We denote this vector field on M by the same letter X. Therefore $\pi_*(X) = \tilde{X}_H \circ \pi$ on K.

We will show that X is a Hamiltonian vector field on M. The function $H : M/\!\!/G \longrightarrow \mathbb{R}$, lifts to a smooth G-invariant function $F : K \longrightarrow \mathbb{R}$. That is $F = H \circ \pi$. Since K is compact in M, by the Tietze-Gleason theorem the function F can be extended to a G-equivariant smooth function on M with compact support. We denote the function by the same letter F. Thus $F : M \longrightarrow \mathbb{R}$ is a smooth G-invariant function with compact support, such that $F = H \circ \pi$ on K. Since X and $\tilde{X}_H$ are π-related and $\pi^*(\omega_{\mathrm{red}}) = i^*(\omega)$, then

$$\iota(X)i^*(\omega) = \pi^*(\iota(\tilde{X}_H)\omega_{\mathrm{red}}) = \pi^*(-dH) = -dF$$

on K. (Recall that $i : K \hookrightarrow M$ is the inclusion map.) Therefore the cohomology class $[\iota(X)i^*(\omega)]$ is equal to zero in $H^1(K; \mathbb{R})$. By the long exact sequence of the pair (M, K) and the hypothesis that $H^1_c(M, K; \mathbb{R}) = 0$, it follows that $[\iota(X)\omega]$ equals zero in $H^1_c(M; \mathbb{R})$. Hence $\iota(X)\omega$ is exact on M. This means that X is a Hamiltonian vector field on (M, ω). Therefore the isotopy induced by the vector field X is a Hamiltonian isotopy.

Thus the vector field X is Hamiltonian with compact support and G-invariant and $X(p) \in T_pK$ for every $p \in K$. That is, X is in the Lie algebra of $\mathrm{Ham}^c_G(M, K)_0$. Thus the 1-parameter group $\{f_t\}$ of Hamiltonian maps induced by X lies in $\mathrm{Ham}^c_G(M, K)_0$. Since X and $\tilde{X}_H$ are π-related it follows that $\Phi(f_s) = h$ for some s.

Since $M/\!\!/G$ is compact and $H^1(M/\!\!/G; \mathbb{R}) = 0$, it follows from a result of Banyaga [2] that the group $\mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}})$ is generated by Hamiltonian maps that are induced by time independent Hamiltonian functions. Therefore Φ is surjective. □

4. Proof of the main result

In general there is no hope that the inequality of the main theorem should hold for for every f in $\mathrm{Ham}^c_G(M,K)_0$. In general we can only have the following estimate.

Proposition 4.7. *For any isotopy* $\{f_t\}$ *in* $\mathrm{Ham}^c_G(M,K)_0$*, with* $f_0 = \mathrm{id}_M$ *and* $f_1 = f$*. Then*

$$\|\Phi(f)\| \leq \mathcal{L}(\{f_t\}).$$

Proof. Let $\{f_t\}$ be a Hamiltonian isotopy in $\mathrm{Ham}^c_G(M,K)_0$ such that $f_0 = \mathrm{id}_M$ and $f_1 = f$. As before let F_t the Hamiltonian function associated to the path $\{f_t\}$. From the proof of Lemma 3.5, we have that $\{\Phi(f_t)\}$ is a Hamiltonian isotopy in $\mathrm{Ham}(M/\!\!/G, \omega_{\mathrm{red}})$ that starts at the identity map on $M/\!\!/G$ with Hamiltonian function $\tilde{F}_t$. Moreover $F_t = \tilde{F}_t \circ \pi$ on K. Thus for each t,

$$\sup_{q\in M/\!\!/G} \tilde{F}_t(q) - \inf_{q\in M/\!\!/G} \tilde{F}_t(q) \leq \sup_{p\in M} F_t(p) - \inf_{p\in M} F_t(p)$$

Hence,

$$\|\Phi(f)\| \leq \mathcal{L}(\{\Phi(f_t)\}) \leq \mathcal{L}(\{f_t\}).$$

□

Proof of Thm. 1.1. Let $\{f^H_t\}$ be the isotopy in $\mathrm{Ham}^c_G(M,K)_0$ generated by H. Then by Prop. 4.7, we have $\|\Phi(f^H)\| \leq \mathcal{L}(\{f^H_t\})$. By Thm. 2.2, the isotopy $\{f^H_t\}$ is length-minimizing, $\|f^H\| = \mathcal{L}(\{f^H_t\})$. It now follows that

$$\|\Phi(f^H)\| \leq \|f^H\|.$$

□

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